One Construction of a Backdoored AES-like Block Cipher and How to Break it

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ESIEA
Operational Cryptology and Virology Lab $(C+V)^O$

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Agenda

Summary of the talk

Introduction

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  - Put a secret flaw at the design level while the algorithm remains public
  - Finding the backdoor must be an untractable problem while exploiting it must be "easy"
  - Historic cases: Crypto AG and Buehler's case (1995)
  - Extremely few open and public research in this area
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  - Extremely few open and public research in this area
  - Known existence of NSA and GCHQ research programs
- Sovereignty issue: can we trust foreign encryption algorithms?

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  - We consider a particular case of backdoors here (linear partition of the data spaces)
- For more details on backdoors and the few existing works, please refer to our ForSE 2017 paper
  - Available on `https://arxiv.org/abs/1702.06475`

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Partition-based Trapdoors

- Based on our theoretical work (Bannier, Bodin & Filiol, 2016; Bannier & Filiol, 2017)
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  - Generalization of Paterson's work (1999)
- BEA-1 is inspired from the *Advanced Encryption Standard* (AES)
  - BEA-1 is a Substitution-Permutation Network (SPN)
  - BEA-1 stands for *Backdoored Encryption Algorithm* version 1

Linear Partitions

Definition (Linear Partition)

A partition of $\mathbb{F}_2^n$ made up of all the cosets of a linear subspace is said to be *linear*.

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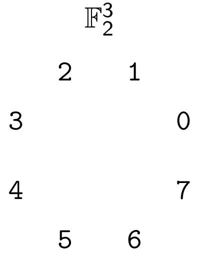

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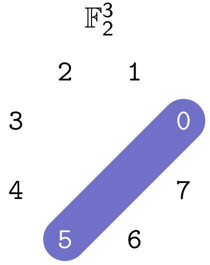

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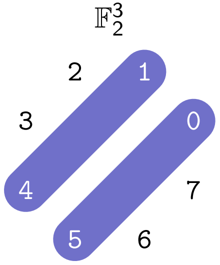

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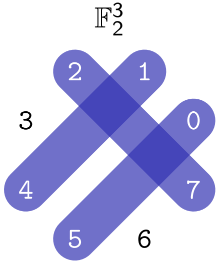

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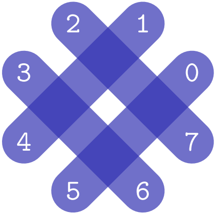

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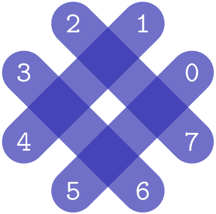

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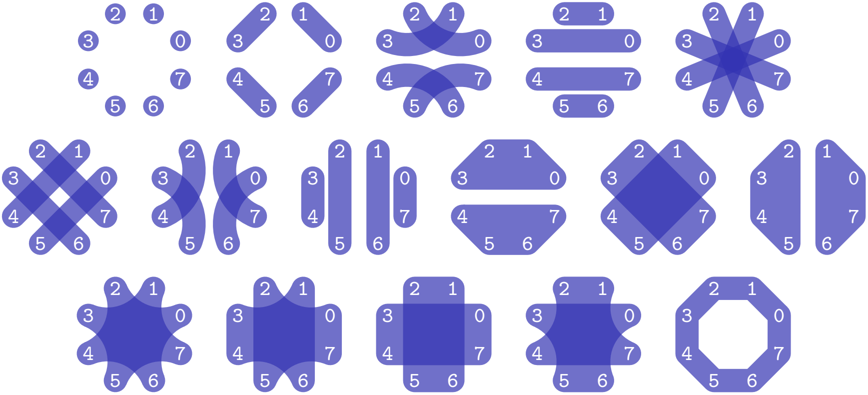

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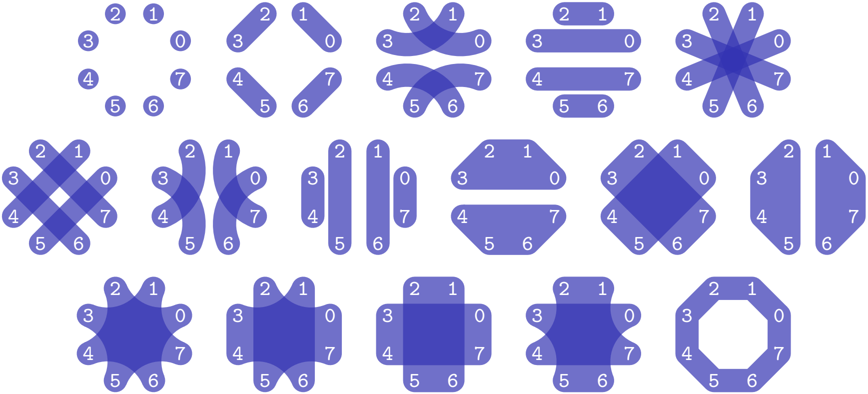

There are 229 755 605 linear partitions over $\mathbb{F}_2^{10}$.

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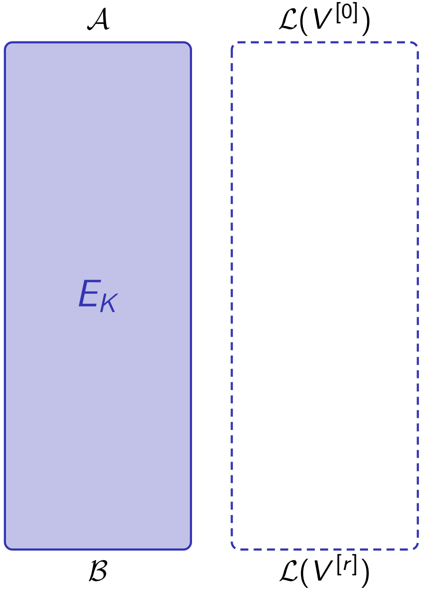

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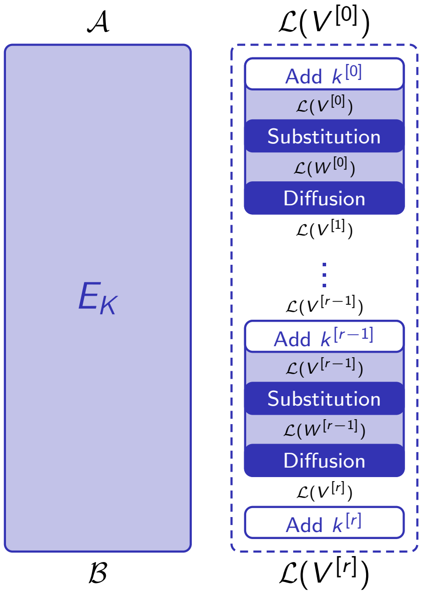

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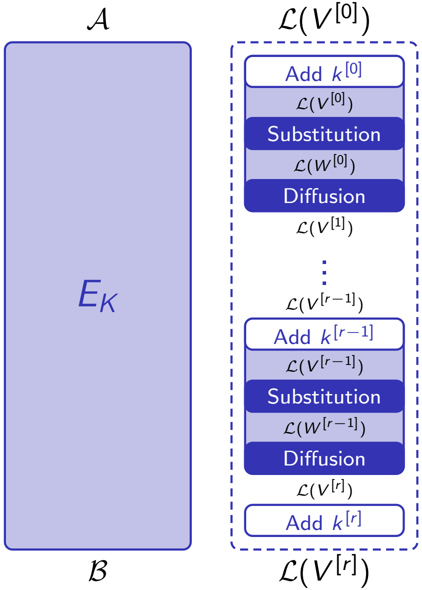
- At least one S-box maps a linear partition to another one.



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- Parameters
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  - Key schedule & key addition (bitwise XOR)
  - Substitution layer (involves four S-Boxes over $\mathbb{F}_2^{10}$)
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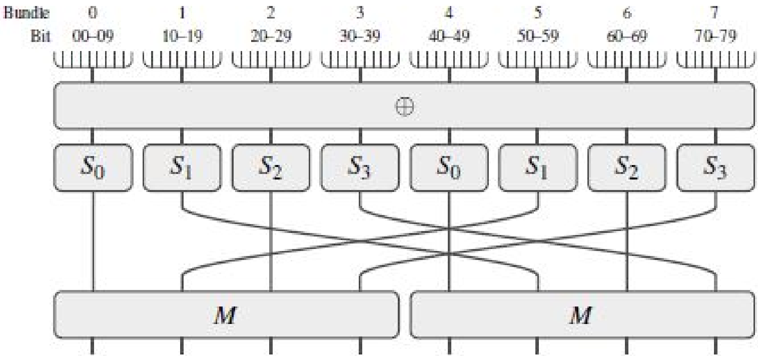
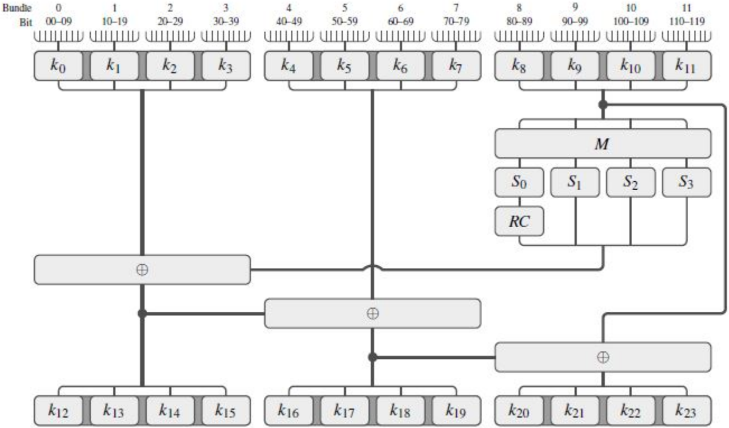
- S-Boxes, linear map M and pseudo-codes for the different functions are given in the ForSE 2017 paper
- BEA-1 is statically compliant with FIPS 140 (US NIST standard) and resists to linear/differential attacks.

BEA-1 Round Function

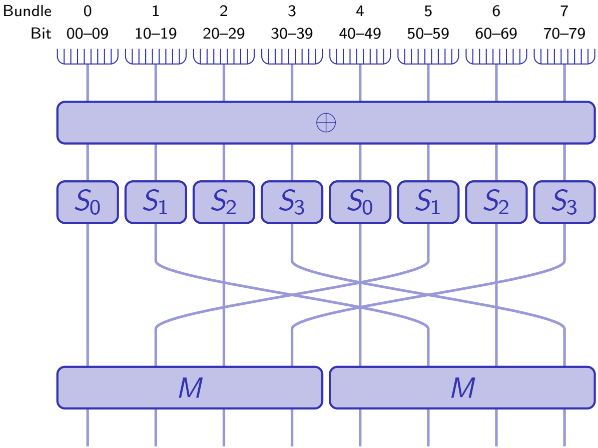


BEA-1 Key Schedule



Summary of the talk

Linear Partitions and the Round Function



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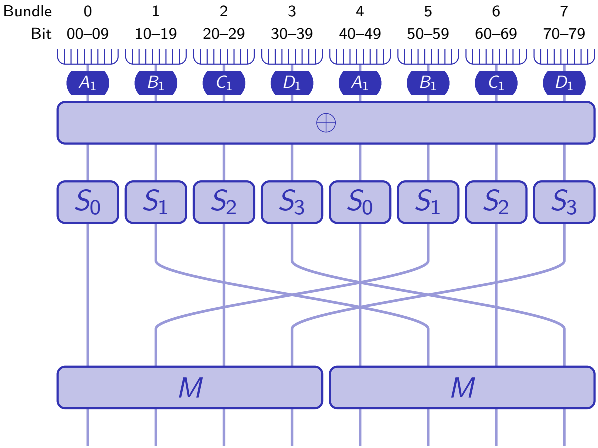

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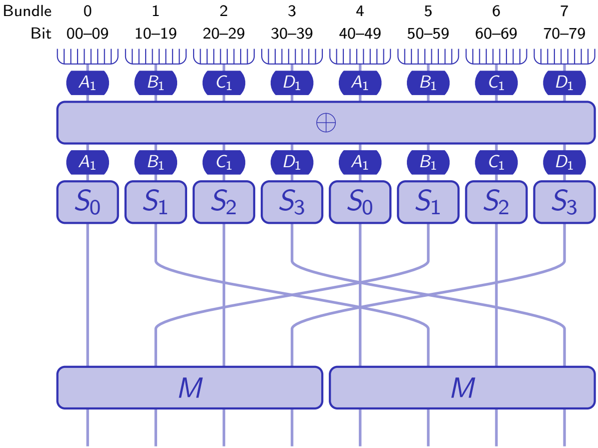

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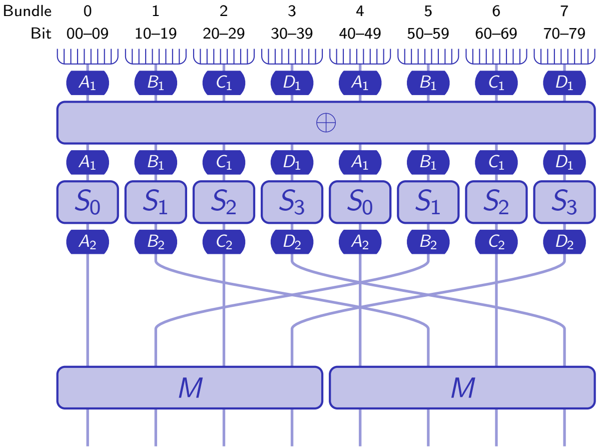

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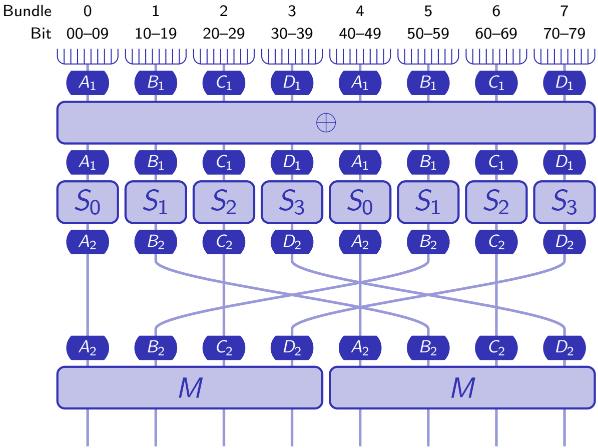

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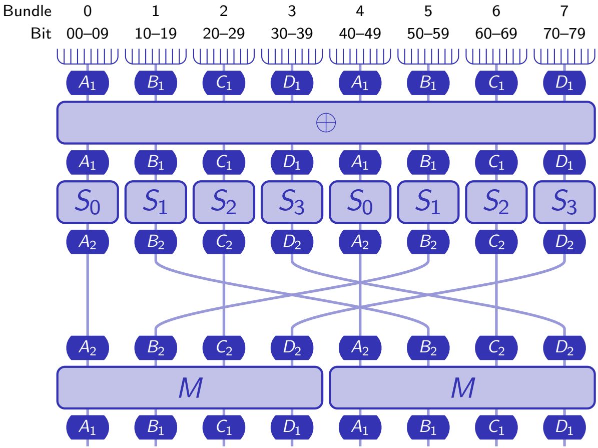

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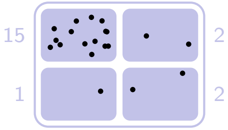

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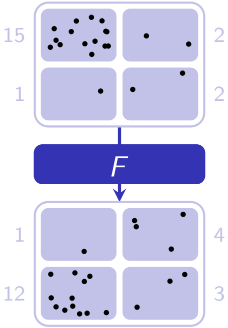

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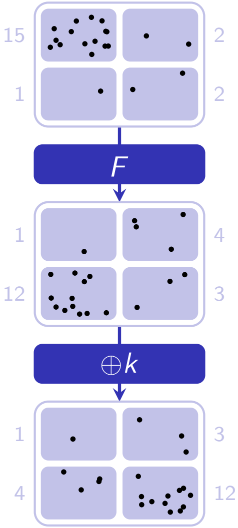

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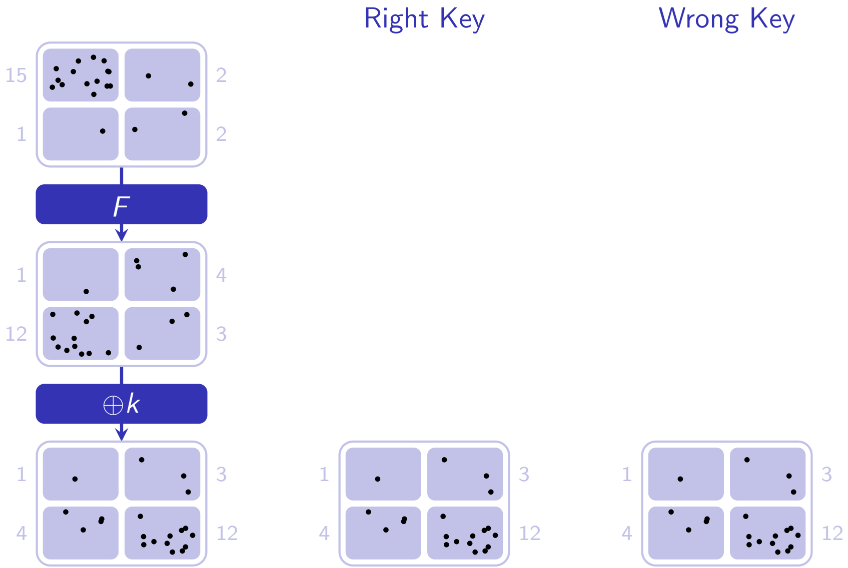

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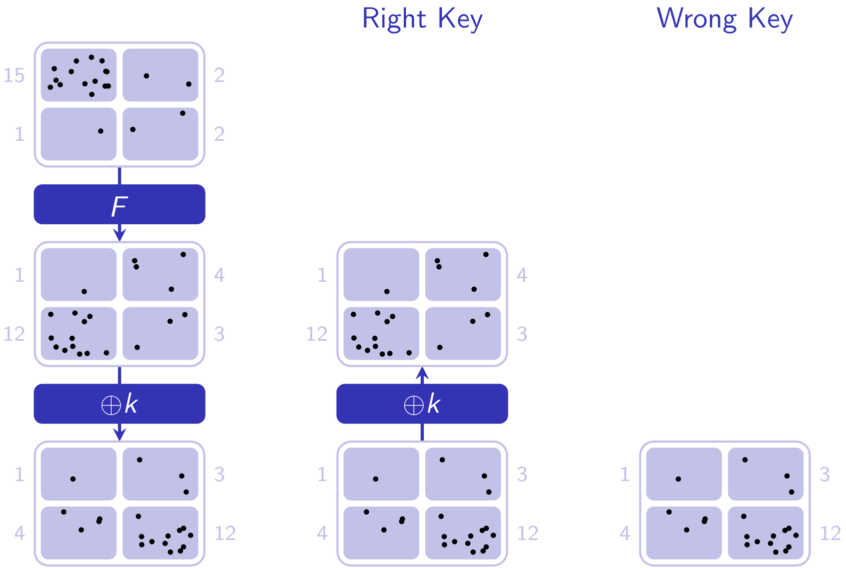

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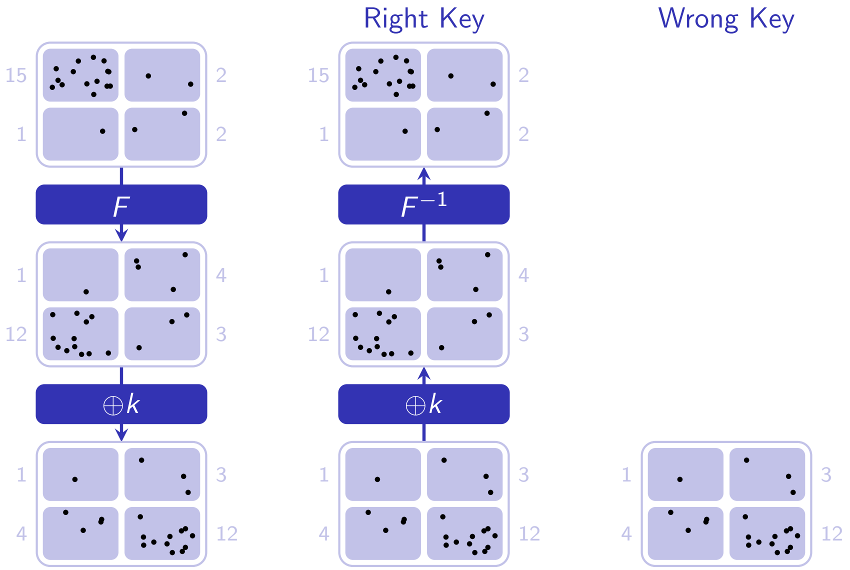

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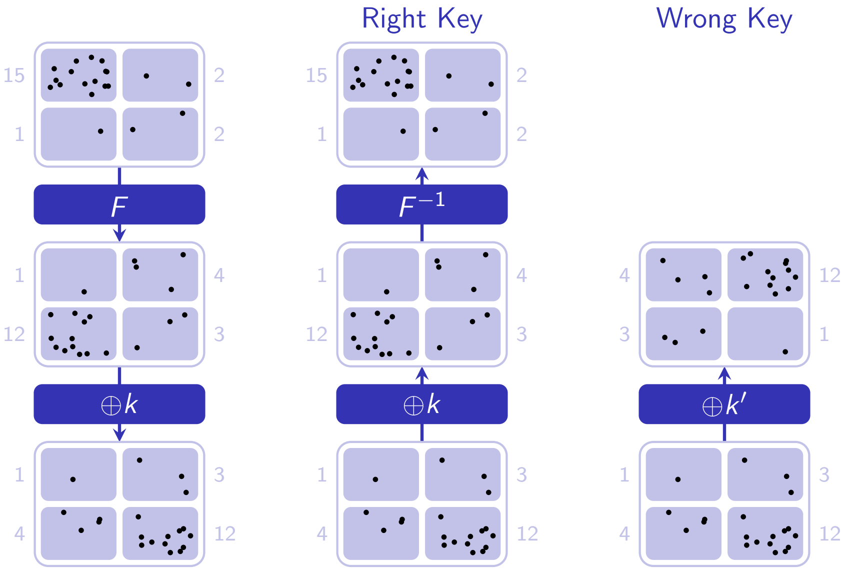

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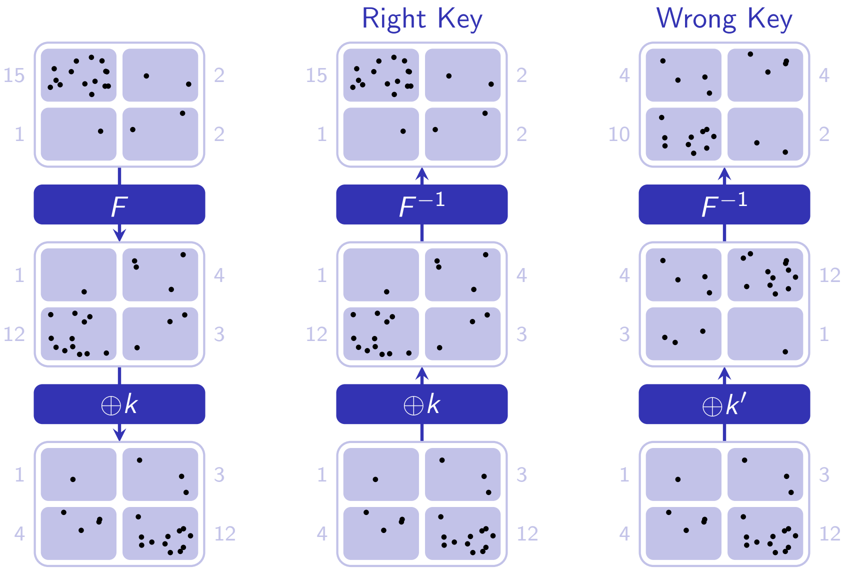
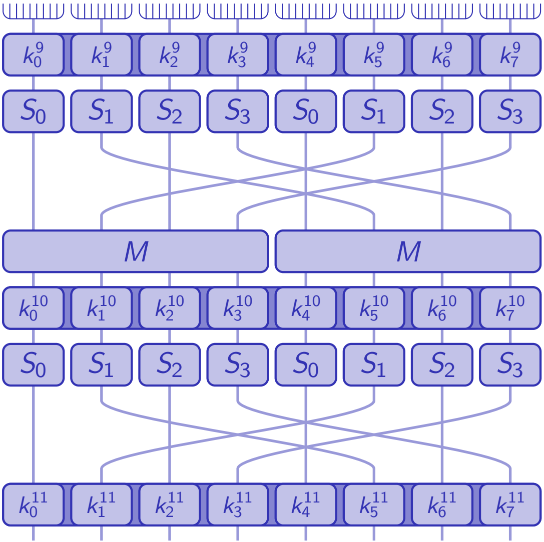


Overview of the Cryptanalysis



Find the output coset of $(A_2 \times B_2 \times C_2 \times D_2)^2$. There are 2^{40} possibilities.

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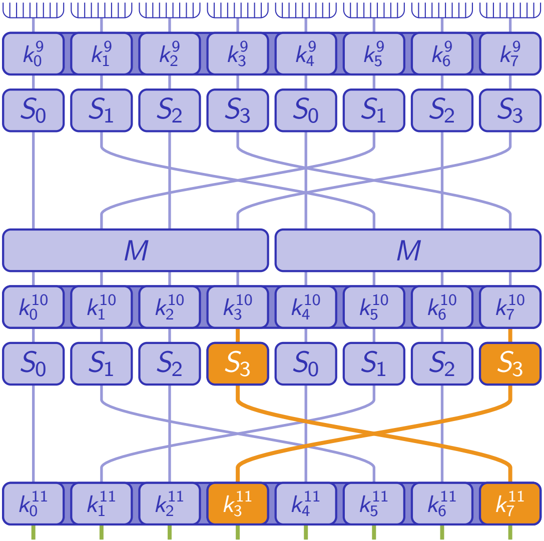

Brute force:

$(k_0^{11}, k_1^{11}, k_2^{11}, k_3^{11}, k_4^{11}, k_5^{11}, k_6^{11}, k_7^{11})$

Test the 2^{15} saved keys:

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At the end of this step, we keep 2^{15} 80-bit candidates from the 2^{80} possible.

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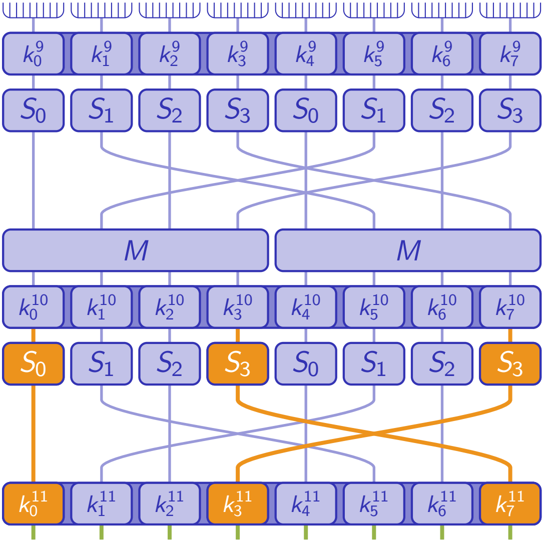

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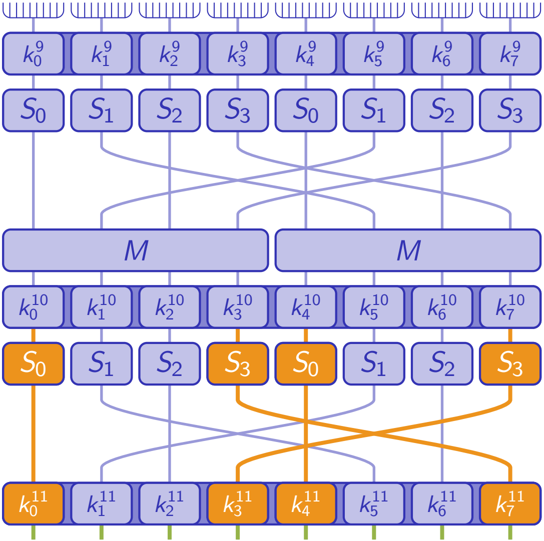

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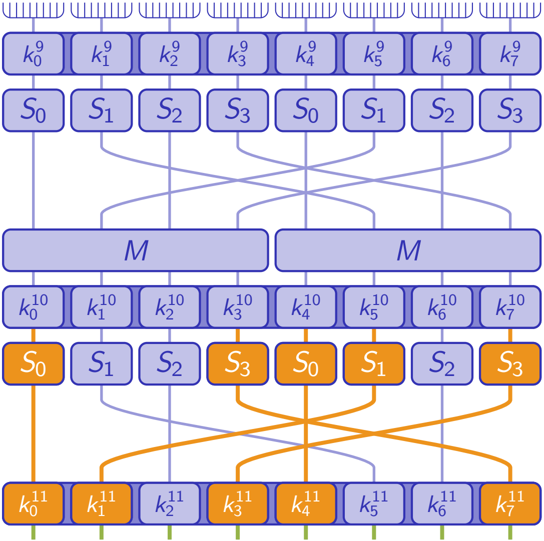

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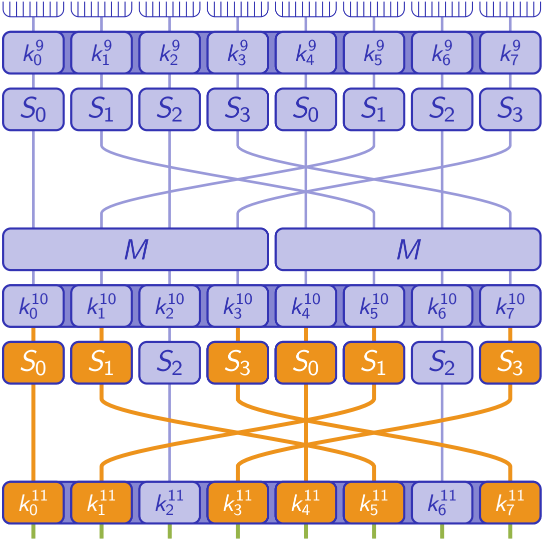

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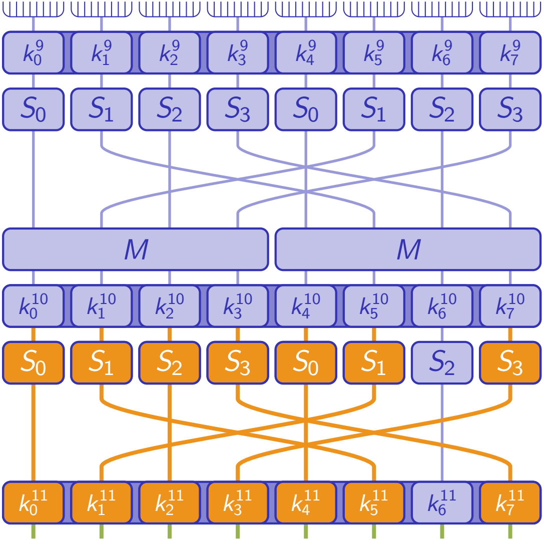

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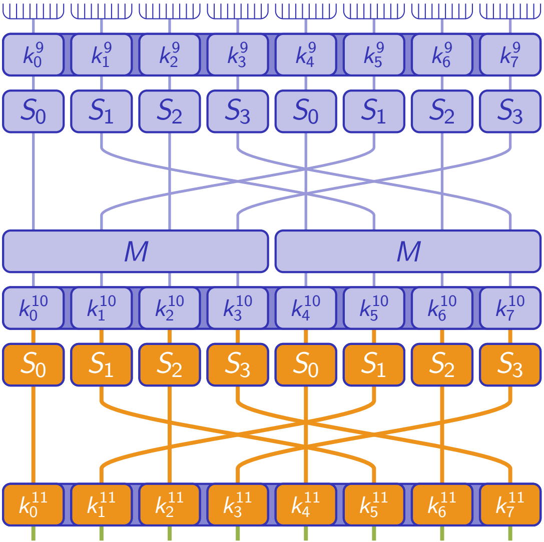

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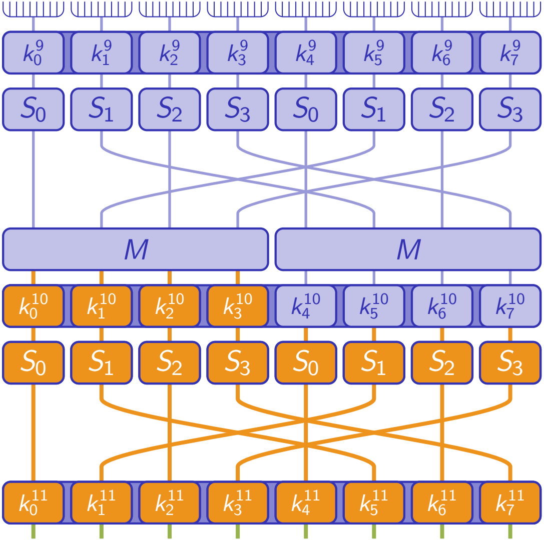


According to the key schedule:

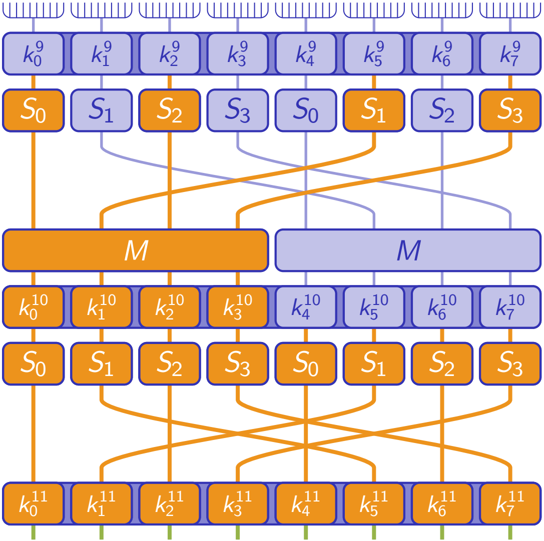
$k_0^{10} = k_0^{11} \oplus k_4^{11}$

$k_1^{10} = k_1^{11} \oplus k_5^{11}$

$k_2^{10} = k_2^{11} \oplus k_6^{11}$

$k_3^{10} = k_3^{11} \oplus k_7^{11}$

Overview of the Cryptanalysis



Test the 2^{15} saved keys:

$(k_0^{11}, k_1^{11}, k_2^{11}, k_3^{11}, k_4^{11}, k_5^{11}, k_6^{11}, k_7^{11})$

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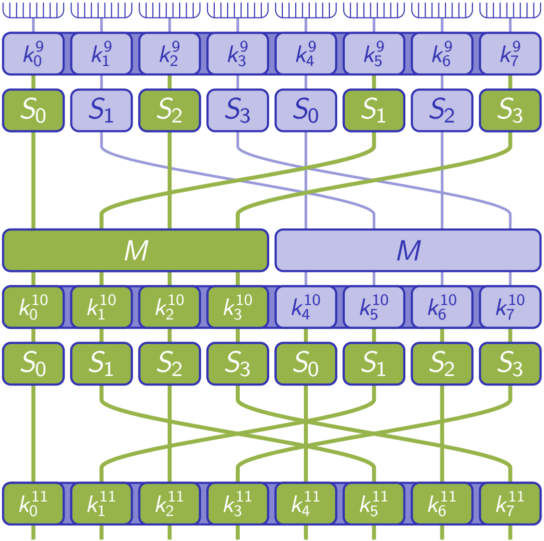
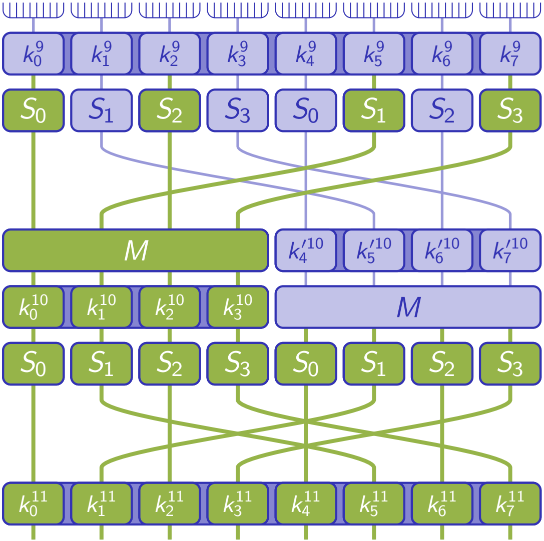


Save the best key:

$(k_0^{11}, k_1^{11}, k_2^{11}, k_3^{11}, k_4^{11}, k_5^{11}, k_6^{11}, k_7^{11})$

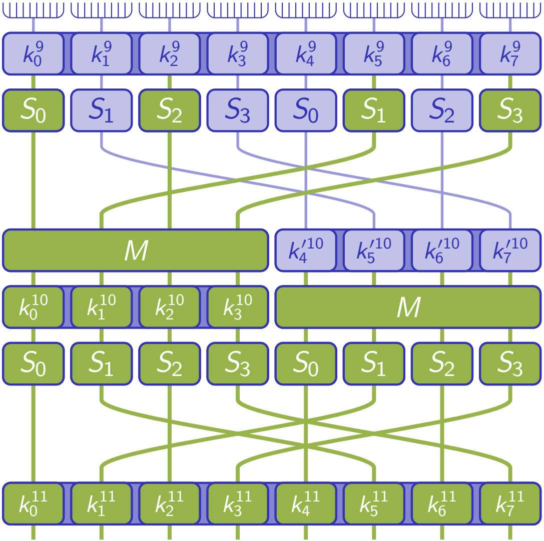
Overview of the Cryptanalysis



Observe that:

$(k_4^{10}, k_5^{10}, k_6^{10}, k_7^{10})$
$= M(k_4'^{10}, k_5'^{10}, k_6'^{10}, k_7'^{10})$

Overview of the Cryptanalysis



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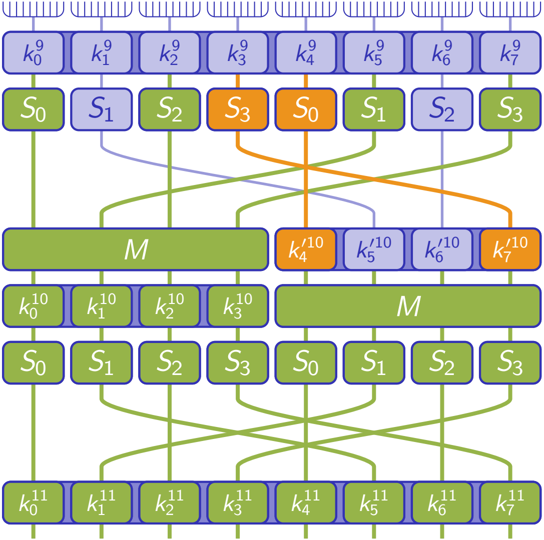

Brute force:

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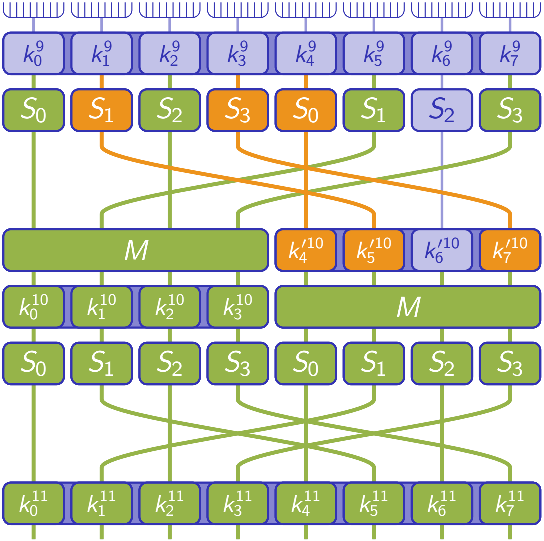

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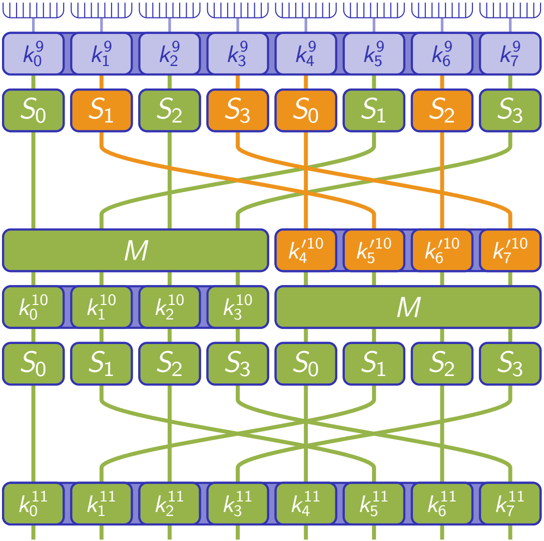

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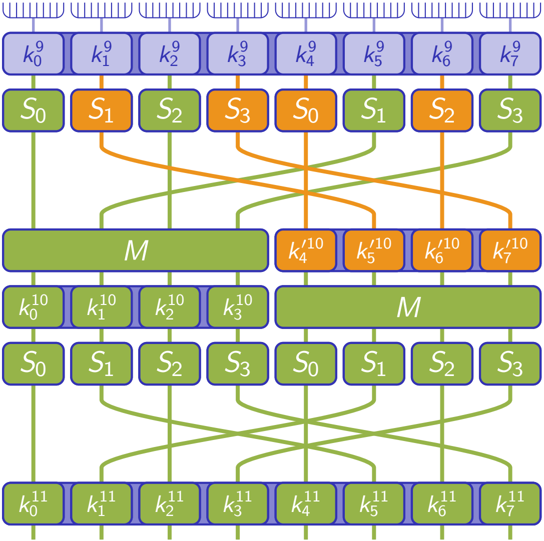

For each saved key,

deduce the cipher key and test it

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Summary of the talk

Conclusion

- Proposition of an AES-like backdoored algorithm (80-bit block, 120-bit key, 11 rounds)
  - The backdoor is at the design level
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  - Illustrates the issue of using foreign encryption algorithms which might be backdoored
- Future work
  - First step in a larger research work
  - Use of more sophisticated combinatorial structures
  - Considering key space partionning
  - Other backdoored algorithms to be published. Use of zero-knowledge cryptanalysis proof

Thank you for your attention
Questions & Answers